# Fitting Liquid Phase Density-pressure Data of monophasic Oil Reservoir Fluid into a Modified Tait Equation (MTE) at 154ºF

**Alagoa, E. E., Egbo, W. Mansi, Bokolo .O. Abovie, & Goldie Jason**
Bayelsa State College of Arts and Science
Elebele P.M.B. 168, Yenagoa,
Nigeria
egbomansi@yahoo.com

***Abstract***
*Liquid phase (monophasic) oil reservoir fluid density-pressure data at 154$^O$F obtained from reservoir fluid analysis (constant composition expansion) report were fitted into a modified Tait equation (MTE) to obtain a model for fluid density computation at pressure range of 2870 to 5000 Psia and constant reservoir temperature of 154$^O$F. The Tait equation parameter constants B and C were determined from regression analysis of the experimental density-pressure data from constant composition expansion study report. The coefficient of determination ($r^2$) was O. 9996 with values of B and C as 11.44psia and 0.0902 respectively. It was observed that, parameter constants B and C depend on the system temperature and components composition in the fluid mixture. The result showed that the modified Tait equation fitted to the density-pressure data at 154$^O$F was able to describe and predict the monophasic homogenous oil reservoir fluid density in the range of pressure with an average absolute percentage deviation of 0.020 when compared to the experimental data.*

***Key words:*** *Liquid phase density-pressure data, monophasic fluid, modified Tait Equation, Linear regression.*

## Introduction

The knowledge of petroleum reservoir fluid density is important and necessary for various applications in the oil and gas industrial process operations. It provides information on fluid pressure gradient, zonal compartmentalization, transition zone characterization and other reservoir fluid quality determination. Oil density is used to determine the commercial quantity of the produce fluid and reserve, it is a critical transport property used in black oil modeling and material balance calculations amongst others (Ahmed, .T. 1989, GUO, B., W.C et al. 2007). Thus, the importance of oil density pressure data cannot be over emphasized in that, oil and gas production process Engineers rely on accurate oil density data to determine flow characteristics, design process equipment that would produce oil well fluids from reservoir to surface conditions, estimate oil reserve and perform material balance calculations. Density is a property that indicates the quality of petroleum and its products and it is a useful property to estimate other fluid properties such as surface tension, viscosity of petroleum (Riazi et.al, 2001) The emergence of high precision densitometers over the years has made accurate measurement of density of liquids at any pressure and temperature possible and quite easier and time saving. Tait equation has been a reliable tool to describe the pressure dependence of pure component compressed liquid densities. However, Tait equation has limited applications to mixtures (Abel Gomes .M F. etal.), that the standard procedure has been to correlate the composition dependence of excess molar volumes at individual pressures via the Redlich-Kister correlation followed by tabulation and publication of the resulting correlation parameters.

Tait equation despite its limited application to mixtures is now being used by many authors to fit density data from PVT experiments that describe the pressure dependence of density of complex fluid mixtures with reasonable accuracy as high as 98.69% (M. Khalil et. al. 2013).

Brief Review of Tait and Tait-like equations for fluid density - compressibility computation
Tait and Tait-like Equations
Tait, P., (1888), proposed his compressibility equation. In 1900, he re-stated his equation in subsequent publications. His original fluid compression equation in literature is given below:

$\frac{Vo-V}{PV0} = \frac{A}{\Pi+p}$ ---------- Equ 1.0

Where $V_0$ is the volume of fresh water at 1 atmosphere and V is the volume of fresh water at higher pressure P,A and n are parametric constants. Tait originally derived his equation for the computation of compressibility of fresh water at high pressure. He fitted his equation to hyperbola (A.T.J. Hayward, 1966).
Many versions of Tait equation have evolved over the decades, some of these equations are stated below.

Tammann, 1895 modified Tait equation is given as follows:
$-\frac{dV}{dP} = \frac{A}{P+B}$ ------------- Equ 2.0
The integrated form of the equation 2.0 is
$V = V_0 \left(1 - A \ln\left(\frac{P+B}{B}\right)\right)$ ------------- Equ 3.0
Other Tait-like equations are also stated as follows:
$\frac{V_0-V}{V_0} = c\log\left(\frac{P+B}{P_0+B}\right)$ --------------- Equ 4.0a
or
$\frac{\rho_0-\rho}{\rho_0} = c\log\left(\frac{P+B}{P_0+B}\right)$ --------------- Equ 4.0b

Where V and ρ are volume and density at pressure P, $V_o$ and $\rho_0$ are volume and density at zero or reference pressure $P_O$, B and C are constants.
B is a temperature dependant while C is a material dependent parameter. Both B and C are obtained by fitting experimental data to Tait equation.
K.M. Shylue paper on compressible two-phsae flow stated that, for baratropic liquids, pressure is a function of the density and the function is expressed in a Tait- like equation of the form;
$P\ (e) = Ae^{\Upsilon} - B$-------------- Equ 5.0
Where A and B are material dependent constants and is the fundamental characterization constant for the liquid.
K, M Shylue reported the values of the constants A,B and ϒ for water and blood to be A= 3001, atm, B= 3000 atm, ϒ=7 and A=614.6 Mpa, B= 614.6 Mpa and ϒ= 5.527 respectively.

Thus A,B and ϒ vary from fluid to fluid.
Carl Schaschke et.al 2013 evaluated the viscosities and densities of different samples of diesel fuels at elevated temperature and pressure where the authors used a Tait-style equation to fit experimental data for fluid compressibility and density prediction. The Tait-style equation is of the form;

$Z = \frac{PVo}{RT}\left[1 - c\log\left(\frac{P+B}{P_{o+B}}\right)\right]$ ----------------Equ 6.0

Where
B and C are the Tait coefficients for the fluid with C being a constant for each diesel fuel sample which deepends on the component composition in the sample, and B is a linear function of temperature, Vo is the molar volume which also depends on temperature, M is a constant.

$V_0 = M/(1000(K_o+K_1T+k_2T^2)$ ---------------------- Equ 7.0 a

$B = bo + b_1T + b_2 T^2$ ------------------- Equ 7.0 b

$K_0$, $K_1$ $K_2$, $b_o$, $b_1$ and $b_2$ are fitting constants
M. Khalil etal. 2013 Measured the densities of super lightweight completion fluid (SLWCF)in the range of (313.15 to 393.15k) and 0.1 to 25 Mpa. The authors modeled a Tait-like equation and fitted experimental data. The predicted densities at the range of temperature and pressure covered by the experiment was said to be 98.69% which was claimed by the authors to be in good agreement with experimental data.

The Tait-like model is of the form;

$E(T,\rho) = \frac{\rho o(T)}{1-Cln\,(\frac{B(T)+P}{B(T)+P_0})}$ ------------- Equ 8.0

Where

$\rho_0\,(T) = A_0 + A_1T + A_2T^2 + A_3T^3$ ………… Equ 8.0a at 0.1 Mpa

$B\ (T) = B_0 + B_1T + B_2T^2$ ……..Equ 8.0b

C is a constant independent of temperature but depends on the molecular composition of mixture if fluid is a mixture.
The authors concluded that, the Tait-like equation can be used to correlate densities of the nontraditional super lightweight completion fluid over a wide range of temperature and pressure with prediction of 98.69% compared to experimental data.
Henrique, F. Costa et.al 2009 correlated the result of PVT measurements of liquid phase aliphatic esters by a Tait equation of the form;

$\rho = \frac{P^{ref}}{1-Cln\,(\frac{P+B}{P^{ref}+B})}$ ……………………….. Equ 9.0

Where $P^{ref}$ is the reference pressure, B and C are constants.
The measured densities of liquid phase aliphatic ester covered the range of temperature was 298 to 393k and up to 35 Mpa pressure.
The authors reported that, the above equation when used to predict the density of the esters using the method proposed by Thomson et. al gave a good result and that, isothermal compressibility can also be calculated from the above Tait equation.
Kumagai and Iwasaki proposed a temperature dependent function of B in Tait equation as,

$B = B_1 + \frac{B2}{T}$ ---------------- Equ.9.0

Where $B_1$ and $B_2$ are parameters obtained from the fitting of Tait equation to the PVT data.
J.H Dymond et. al, and R. Malhotra. 1988, stated that, Tait equation is widely used to fit liquid density data over wide range of pressure. Over the years, various modifications of the original equation of Tait have emerged. Isothermal density- compressibility for homogenous liquids up

to elevated pressures and temperature are widely represented by the following Tait-like equation.

$$\frac{\rho-\rho_0}{\rho} = c\ln\left(\frac{P+B}{P0+B}\right) \text{------------ Equ 10.0a}$$

In terms of volume,

$$\frac{Vo-V}{V0} = c\ln\left(\frac{P+B}{P0+B}\right) \text{------------ Equ 10.0b}$$

Where the subscript o refers to low pressure usually I atm or bubble point pressure and B and C are parameters obtained from fitting experimental data.

Dymond and Robertson (1982) presented density data for pure hydrocarbons such as octane, decane, and dodecane and their 50% binary mixtures at four different temperatures from $25^0$c to $100^o$c and pressures from 1 atmosphere to 500 Mpa. To calculate isothermal densities over the pressure range, the authors applied secant bulk modulus k as a polynomial function of pressure as follows

$$K = K_0 + ap + bp^2 \text{ ------------- Equ 11.0}$$

Where K is defined as a function of pressure and density as follows:

$$K = \left(\frac{P+Po}{\rho-\rho 0}\right)\rho. \text{ ---------------- Equ 11a.0}$$

a,b and $k_o$ are presented in tabular form in literature for all single components and binary mixtures.

The corresponding isothermal density can then be calculated from;

$$\rho = \left(\frac{\rho_0 K}{K-(P-P0)}\right) \text{ ----------------- Equ 12.0}$$

Where $\rho_0$ is the density at 101. 325 Kpa and 298.15k.

The Tait equation is considered to be the most satisfactory of the equations investigated in reproducing liquid density measurements over a wide range of pressure (Dymond and Malhotra, 1987, 1994).

Dymond and Malhotra (1987) applied Tait equation to correlate the density data from different sources (Dymond et.al, 1980 Dymond and Robertson, 1982, Kashiwagi and Makita, 1982, and Doolittle, 1964) on n- alkanes from n- hexane up to n- heptadecane with the C parameter as constant equal to 0.02 and the B parameter as a function of reduced temperature as;

$B = 341.539-734.292\ T_r + 411.189\ T_r^2-(C_n-6)$ – Equ 13.0

Where $C_n$ is the characteristic carbon number which is the actual number of carbon atoms in n-alkanes.

This paper employed simple linear regression of Density-pressure data from PVT studies to obtain a Tait-like model capable of describing liquid phase density pressured data at $154^0$F.

## 2.0 Methodology of fitting experimental data into modified Tait equation

Pressure – density data obtained from reservoir fluid analysis were fitted into a modified Tait equation of the form; as stated earlier in equations 4.0a and 4.ob:

$$\frac{VO-V}{VO} = \frac{\rho-\rho o}{\rho} = c\log\left(\frac{P+B}{PO+B}\right)$$

Liquid phase density data for a single phase homogenous reservoir fluid sample from PVT oil study report were used to correlate a Tait – like equation as stated above. Table 1.0 shows the PVT study report of an oil reservoir fluid sample at $154^0$F.

**1.0 Liquid phase density of petroleum reservoir fluid during constant com position expansion (CCE) Study at $154^oF$.**

| Pressure (Psia) | Liquid phase Density (g/cc) | Relative Volume |
|---|---|---|
| 5000 | 0.7533 | 0.9782 |
| 4500 | 0.7500 | 0.9826 |
| 4000 | 0.7463 | 0.9874 |
| 3500 | 0.7424 | 0.9926 |
| 3000 | 0.7381 | 0.9984 |
| +2870 | 0.7369 | 1.0000 |

+ Bubble point pressure ($P_0$ = Reference Pressure).

## 2.1 Transformation of modified Tait equation into linear equation for linear regression analysis.

$$\frac{VO-V}{VO}=\frac{\rho-\rho o}{\rho}=c\log\left(\frac{P+B}{PO+B}\right)$$

Where,
Vo= Reference fluid volume at pressure $P_o$
V =fluid volume at pressure P.
B and C are the usual Tait constants.

+ Bubble point pressure ($P_0$=Reference pressure).

Let $\frac{\rho-\rho o}{\rho}=c\log\left(\frac{P+B}{PO+B}\right)$

Let $Y=\frac{\rho-\rho o}{\rho}$ be the fractional change in fluid density.

Let X = ln(P+B) ………………….. Equ14.0

The linear form of the modified Tait equation given by;

Y= a + bX ……………………. Equ 15.0

Where $a=-\frac{C}{ln10}\ln(Po+B)$……….Equ 15.oa

$b=\frac{C}{ln10}$ ………………….Equ15.0b

a and b are linear regression constants which determine the Tait
Constants, B and C in Equations 15.0 a and b respectively.

## 2.2 Linear regression of Density – pressure data of table 1.0 to determine regression coefficients and Tait constants.

From Eqn 15.0;

Let $\sum Y = na + b\sum x$ ----------- Equ16.0

$\sum XY = ax + b\sum x^2$ ------------ Equ17.0

Solving Equation (16.0) and (17.0) simultaneously, we have;

$$a = \frac{\sum Y \sum X^2 - \sum X \sum XY}{n\sum X^2 - (\sum X)^2}$$

$$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

and $r = \frac{n\sum XY - \sum X \sum Y}{\sqrt{(n\sum X^2 - (\sum X)^2)(n\sum Y^2 - (\sum Y)^2)}}$

For n = 6, a, b and r are determined from the entries in table 2.0

**Table 2.0 regression data analysis**

| $(\frac{\rho - \rho_o}{\rho})$ ( Y) | In(p) (X) | ( $Y^2$) | ($X^2$) | (XY) | | |
|---|---|---|---|---|---|---|
| 0.021770 | 8.51719 | 0.0004739 | 72.54253 | 0.1854192 | | |
| 0.017470 | 8.41183 | 0.0003052 | 70.75888 | 0.1469547 | | |
| 0.012600 | 8.29405 | 0.0001588 | 68.79127 | 0.1045050 | | |
| 0.007408 | 8.16052 | 0.0000549 | 66.59409 | 0.0604531 | | |
| 0.001626 | 8.00637 | 0.0000026 | 64.10196 | 0.0130184 | | |
| 0.000000 | 7.96207 | 0.0000000 | 63.39456 | 0.0000000 | | |
| $\sum Y$ 0.060874 | $\sum X$ 49.35203 | $\sum Y^2$ 0.0009954 | $\sum X^2$ 406.18329 | $\sum XY$ 0.5103504 | | |

a $= \frac{\sum Y \sum X^2 - \sum X \sum XY}{n\sum X^2 - (\sum X)^2} = \frac{(0.060874)\ (406.18329) - (0.5103504)(49.35203}{6\ (406.18329)\ -(\ 49.35203)2}$

$= -0.31203$

b $= \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)2} = \frac{6\,(0.5103504) - (49.35203)(0.060874)}{6\ (406.18329)\ -(\ 49.35203)^2}$

$= 0.03917$

r $= \frac{n\sum XY - \sum X \sum Y}{\sqrt{\{(n\sum X^2 - (\sum X)^2)(n\sum Y^2 - (\sum Y)^2)\}}}$

$$=\frac{6(0.5103504)-(49.35203)(0.060874)}{\sqrt{\{6\left((406.18329)-(49.35203)^2\right)\left(6(0.0009954)-(0.060874)^2\right)\}}} = 0.99978$$

The coefficient of determination ($r^2$) is 0.99960.
The Tait constants B and C are obtained from the equations 15.0a and 15.0b as follows:

$-0.31203 = -\frac{C}{ln10}\ln(P_0+B)$ and $0.03917 = (\frac{C}{ln10})$,

where C = 0.0902, $\ln(P_0+B) = \frac{0.31203}{0.03917}$,

$P_0 + B = \exp(\frac{31203}{3917}) = 2881.44$,

$B = 2881.44 - P_0$

When reference Pressure = 2870 Psia),

B = 2881.44 - 2870 = 11.44 Psia

The values of B and C can now be substituted into the modified Tait model in equation (10.0b) to obtain the model of the form:

$$\frac{\rho-\rho_0}{\rho} = \frac{0.0902}{ln10} \ln\left(\frac{P+11.44}{2881.44}\right) \text{ ..................... Equ18.0}$$

Hence, the modified Tait Equation fitted for the pressure- density data at $154^0$F is given by

$$\rho\ (P)_{T=\text{constant}} = \frac{\rho_0}{1-0.03917\ln\ (P_0+11.44)} \text{ ............Equ19.0}$$

Where P = fluid pressure.
$\rho_o$ = reference density at bubble point pressure (0.7369g/$cm^3$).
$\rho$ = Density at pressure P above the bubble point at temperature T.
T = constant temperature of fluid ($154^0$F).

**Table 3.0 comparison of pressure–Density values calculated from the modified Tait model with experimental Data.**

| S/N | Pressure(Psia) | Liquid phase density (g/cc) Experimental data | Liquid phase density calculated from the modified Tait equation19.0 | Absolute Deviation |
|---|---|---|---|---|
| 1. | 5000 | 0.7533 | 0.7532 | 0.0001 |
| 2. | 4500 | 0.7500 | 0.7501 | 0.0001 |
| 3. | 4000 | 0.7463 | 0.7466 | 0.0003 |
| 4. | 3500 | 0.7424 | 0.7427 | 0.0003 |
| 5. | 3000 | 0.7381 | 0.7382 | 0.0001 |
| 6. | 2870 | 0.7369 | 0.7369 | 0.0000 |

Average absolute Deviation 0.00015

**3.0 Result and discussion**

Pressure –density experimental data at $154^0$F were fitted into a modified Tait Equation using linear regression technique. Table 2.0 showed the regression Analysis of experimental data of table 1.0. The coefficient of determination for the data was 0.9996 which indicated a very strong correlation of pressure with Density. The Tait constants B and C were determined from the regression analysis result in table 2.0.

The modified Tait equation was used to calculate densities at pressure range of 2870 to 5000 Psia at constant temperature of $154^0$F with an average absolute deviation of 0.00015 and percentage average absolute deviation of 0.020% from experimental data. This showed that, the modified Tait equation can be used to model liquid phase density of any homogenous liquid phase fluid system which predicted the experimental data with a reasonable accuracy of 99.96%.

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